*Orbis Idearum*, Volume 8, Issue 2 (2020), Pages 11-35
ISSN: 2353-3900

# TOPOLOGICAL REASON AND SURROUNDING IDEAS IN GASTON BACHELARD AND KURT GÖDEL

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ABSTRACT

This short piece aims to clarify, in the light of a comparative methodology, some aspects of the philosophical topology put forward by Gaston Bachelard in one of his final epistemological works, with the objective of making clear one of the many 'philosophical souls' – to use an expression of Moritz Schlick – which can be found nestled in the sciences and in particular in the mathematical corpus of his time. At the same time, this piece aims to show how this need of a topology or topological reason finds analogy with that path of a more philosophical nature that characterized the final activities of Kurt Gödel in the 1950's, as can be seen in his rich, articulate and now available *Nachlass* – his attempt to classify the different philosophies of mathematics. This makes it possible to show that different paths, despite having emerged in cultural contexts far from each other, but all oriented towards understanding the 'reasoning of science' of their time – to use an expression of Federigo Enriques – sometimes come to very similar results, such as the need for a new non-analytical philosophy of mathematics.

KEYWORDS: Gaston Bachelard, Epistemology, Topology, Kurt Gödel, Philosophy of Mathematics

*True philosophy is nothing other than the spirit of science*
Maximilien Winter

*As far as the 'building of science' is concerned,*
*it is possible to construct it without first laying foundations*
Gaston Bachelard

*Logical thought is not offered to us easily,*
*and can't produce its most beautiful fruit on its own plant.*

Received 15 October 2020, Revised 20 October 2020,
Accepted 3 November 2020, Available online 21 December 2020

*It needs to be engaged with something*
*other than itself in order to be truly productive*
Hélène Metzger

Almost all the critical studies of the work and thought of Gaston Bachelard dedicate a certain space to the *philosophical topology*, as presented in the first pages of the 1949 work, *Le rationalisme appliqué*[1] – as it is not something commonly found in the works of other philosophers and in the philosophy of science in particular; but it must be pointed out immediately that this *topology* is only one among the many other original theoretical methods advanced by the French epistemologist in order to comprehend the philosophical dimension implicit in the sciences of the early twentieth century, and especially in the new physics, in which mathematization processes were coming to play an ever more important role. On the one hand, the *topology* represents the result of a series of reflections which had already started in 1928[2], beginning with *Essai sur la connaissance approchée*, as it has been shown in a recent study containing epistemological contributions on the cognitive structure of maths and the process of abstraction, with analyses of the works of Bernhard Reimann, Hermann Weyl[3] and of *Analysis Situs*; at the same time, the *topology* cannot be explained adequately if one does not

[1] G. BACHELARD, *Le rationalisme appliqué*, P.U.F., Paris 1970[4], p. 7.

[2] As is known, but it is always opportune to restate it, 1928 is a decisive year for the philosophy of science. Next to Carnap's *Der Logische Aufbau der Welt*, we must also put this Bachelardian work, which constitutes yet another research strategy, of epistemological nature, on the conceptual structure of mathematics.

[3] The attention to the work of Riemann and Hermann Weyl, an uncommon thing in 20th century philosophy of science as Ludovico Geymonat denounced in *Filosofia e filosofia della scienza* (Feltrinelli, Milano 1960, p. 185), has determined the direction taken by French epistemology, in particular Bachelard's. On this cfr. C. ALUNNI, *Hermann Weyl chez Gaston Bachelard*, in C. ALUNNI, M. CASTELLANA, D. RIA et A. ROSSI (cur.), *Albert Einstein et Hermann Weyl 1955-2005. Questions épistémologiques ouvertes*, Barbieri-Selvaggi Ed., Manduria 2010, pp. 13-24 and ns. *Sur un petite phrase de Riemann. Aspects du débat français autour de la Reasonable Effectiveness of Mathematics*, "Revue de synthèse", t. 138, (2017), pp. 195-229. The critical literature on Bachelard has sometimes given a certain space to the role assigned to mathematics, but has not considered it constitutive of his most original propositions; on the other hand more recent works have highlighted its strategic role, also for the constant attention to the processes of abstraction taking place therein; cfr. the essays contained in C. ALUNNI, cur., *Philosophie et mathématique. Bachelard et les mathématiques*, "Revue de synthèse", t. 136, (2015) and *Spectres de Bachelard. Gaston Bachelard & l'École surrationaliste*, Hermann, Paris 2018; G. IENNA, *Presentazione* to G. BACHELARD, *Metafisica della matematica*, trad. it. by C. Alunni and G. Ienna, 2016, Castelvecchi, Roma 2016, pp. 5-18 and F. PALOMBI, *L'elogio dell'astrazione. Gaston Bachelard e la filosofia della matematica*, Mimesis, Milano 2017 where chapter IV is entirely dedicated to the topology and its genesis in Bachelardian discourse.

keep in mind that it is above all the result of a remarkable and constant reflection, almost unique in its kind, on the conceptual structure of mathematical physics as it came to take shape first in the work of Einstein and then in the quantum mechanics of Paul Dirac[4]. On the other hand the *topology* comes to constitute, as occurs in every Bachelardian work, almost a chapter in itself thanks to its investigating with ever more appropriate instruments that which the Italian mathematician and epistemologist Federigo Enriques – whom Bachelard nods to both in *Le nouvel esprit scientifique* and *La formation de l'esprit cientifique* of 1938 – was already calling, in the early 20th century, 'the reasoning of science' and its 'implicit philosophy'[5]. Bachelard's scholars[6] have not always sufficiently underlined the not-coincidental fact that the first chapters of Bachelard's works are expressly planned to outline the need for a philosophy that could measure up to the contents of the *nouvel esprit scientifique,* a philosophy which would make explicit the *esprit*'s various and differently articulated dimensions without the obligation to operate a *reductio ad unum* – an obligation which must always be negotiated with when we legitimately want to grasp the basic characteristics of the cognitive processes as they are put in action by the various sciences. It is no coincidence, then, that in all of Bachelard's writings a strategic role is assigned to the historical-epistemological investigation, considered an essential instrument for entering the *tissu vivant* of sciences and to systematically reach their *avant-postes*; such a theoretical strategy was employed to avoid normative discourse from two camps: from philosophers, accused of ignoring the extreme plurality of scientific facts; and from scientists, who often use approaches which are not appropriate to the scope of the *enjeux* that they are implementing[7].

[4] Whether in the recently re-edited *La valeur inductive de la relativité* of 1929, or in 1934's *Le nouvel esprit scientifique*, there are profound and original analyses on the role of tensor calculation, like in *La philosophie du non* and *L'activité rationaliste de la physique contemporaine* of 1951 where the scientific work of Dirac is analysed; and on this cfr. C. ALUNNI, *Relativités et puissances spectrales chez Gaston Bachelard*, "Revue de synthèse", n. 1 (1999), pp. 73-110 and nss. *Razionalismi senza dogmi. Per una epistemologia della fisica matematica*, Rubbettino Ed., Soveria Mannelli 2004 and *Gaston Bachelard ou la rêverie anagogique dans les enjeux du surrationnel*, "Revue de synthèse", t. 136 (2015), pp. 93-116.

[5] F. ENRIQUES, *Scienza e razionalismo*, Zanichelli, Bologna 1986[2], chap. I and pp. 114-115.

[6] An exception to which is the important study of C. VINTI, *Il soggetto qualunque. Gaston Bachelard fenomenologo della soggettività epistemica* ,Edizioni Scientifiche Italiane, Napoli 1997 and the more recent Ch. ALUNNI, *Spectres de Bachelard*, cit.

[7] The terms *avant-postes* and *enjeux*, like so many others, make up part of the rich vocabulary intentionally employed by Bachelard to situate himself at the heart of scientific questions. As affirmed by Alunni in his works herein cited, in the rich but at times misleading critical literature on the French epistemologist there is still a lack of an organic study on such linguistic aspect – which, among other things, would make clearer the original and undeniable aspects of his philosophy of science, not recognized in its true dimension.

With a language which has irritated and continues to irritate not only epistemologists belonging to other important traditions of research, but also scientists like René Thom, who denounce the literary deviation suffered by his epistemology, Bachelard employs the then used expression *philosophie scientifique*[8], while also introducing other clarifying terms of such an expression, such as *philosophie ouverte, philosophie dispersée, philosophie distribuée, philosophie différentielle, philosophie dialoguée, poliphilosophie, philosophie du mobile* and *philosophie du détail*[9], above all in order to avoid antiquatedly positivist and scientist descriptions of science. These expressions, dense on the intellectual level in that they signify real programs of research, and present above all in *La Philosophie du non* (1940), prepare the ground for the next phase of investigation, that of 'philosophical topology', helping us to better understand its aims and underlying epistemic sense. For this reason these terms take on the task of eliminating all pretensions of a totalizing vision, or of what he calls the "integral philosophy of philosophers", against which even some scientists uncritically put forward their own philosophical point of view, derived exclusively from a unilateral vision of the "realm of facts"[10]; these unilateral positions then produce, and are at the same time victims of, real "epistemological obstacles" to the comprehension of the veritative contents implicit in different moments of the history *de la pensée des sciences* – another idea, not to say category, of the Bachelardian path of research, little studied in its true dimensions[11] and ever more relevant

[8] Recall that in 1935 and 1937 there took place in Paris, in the wake of those in the first decade of the century, two significant *Congrès de Philosophie Scientifique* which saw the establishment of logical neo-positivism and the development of that same philosophy of science, and on this cfr. *Alle origini della 'nuova epistemologia'. Il Congrès Descartes del 1937*, Il Protagora, Lecce 1992.

[9] All these terms, however, acquire a more precise meaning if they are inserted in that 'dialectic suprarationalism' enucleated in *La philosophie du non*, which is a project of dynamic and plural rationalism that takes account for the various 'regions' of scientific thought with the recognition of individual epistemic values. On this cfr. *Il surrazionalismo di Gaston Bachelard*, Glaux Ed., Napoli 1974 and *Razionalismi senza dogmi. Per una epistemologia della fisica matematica*, cit., chap. VI; Ch. ALUNNI (cur.), *Science et philosophie au XX^e^ siècle. L'École de Zürich et le programme surrationaliste*, "Revue de synthèse", t. 126, (2005/29) and *Spectres de Bachelard*, cit. ; cfr. also V. BONTEMS, (cur.), *Bachelard et l'avenir de la culture. Du surrationalisme à la raison créative*, Presses des Mines, Paris 2018.

[10] G. BACHELARD, *La philosophie du non*, P.U.F., Paris 1970[5], p. 7. With the language of the Columbian writer Nicolás Gómez Dávila, it can be said that both these positions, on the one hand of the philosophers and the other of the scientists, are dominated by "imperialistic pretensions", those which an articulate and healthy epistemology – in Bachelardian terms dialectic and historical, that is more appropriate to the real scientific contents – must be able to identify and eliminate as much as possible. Also cfr. N. GÓMEZ DÁVILA, *In margine a un testo implicito*, trad. it. F. Volpi, Adelphi, Milano 2001, p. 16

[11] Only in the work of Charles Alunni – starting from the first years of this century with the foundation of a Laboratoire Disciplinaire very significantly named "Pensée des sciences" –

and indispensable to be able to combat with appropriate weapons the various positions which lead to those which Georges Canguilhem has called 'scientific ideologies'[12].

Keeping in mind such a fundamental idea of scientific thought[13] prevented Bachelard (it is not a coincidence that Dominique Lecourt will define Bachelard as "*the great forgotten*, *by both scientism and anti-science*"[14]) from falling into normative positions, as often happened to other, more studied and practiced strands of epistemological thought, those which are more involved in trying to establish supra-historical criteria of scientific philosophy. And if in his works there appear geometric representations like the notion of epistemological profile and spectrum[15] as can be found in *La philoso-*

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has such an idea been well emphasised and put into practice; but the idea of scientific thought found its own more precise collocation in the 1934 work of Federigo Enriques, *La significazione de l'histoire de la pensée scientifique*, not coincidentally kept systematically present by Bachelard. On this cfr. thenss. *Il metodo storico in filosofia della scienza*, in F. ENRIQUES*, Il significato della storia del pensiero scientifico*, Barbieri-Selvaggi Ed., Manduria 2007, p. 87-127 and *Il tetraedro storico-epistemologico*, in F. ENRIQUES-H. METZGER, *Storia e struttura del pensiero scientifico*, Barbieri-Selvaggi Ed., Manduria 2014, pp. 117-145.

[12] Cfr. G. Canguilhem, *Ideologia e razionalità nella storia delle scienze della vita*, trad. it., La Nuova Italia, Firenze 1992, pp. 25-38; it must be kept in mind that in the 20th century French epistemological culture the attention to the ideological implications of scientific theories was a not inconsiderable characteristic precisely due to the anchorage to their historical dimension, and in this sense the figure of Hélène Metzger distinguished herself in her incomplete and posthumous work, *La science*, *l'appel à la religion et la volonté humaine*, written during the years of the Nazi occupation of France. cfr. H. METZGER, *La scienza, l'appello alla religione e la volontà umana*, trad. it. by M. Castellana, Pensa Multimedia-ENS 'Pensée des sciences', 2014.

[13] The Bachelardian term *pensée des sciences* is translated as 'scientific thought', but it must be remembered that this latter, commonly used term makes part of the lexicon of the history of science, a discipline which in the 1930s was, just like the philosophy of science, a field still in its formative stages, and one which in different circles developed in alternative to the philosophy of science itself; but, to use an expression of Bruno Widmar, they are both 'conceptual techniques' directed towards accounting for the 'reasoning of science' on the theoretical as well as the historical plain: cfr. B. WIDMAR, *L'epistemologia,* (1974), by M. Castellana, Milella, Lecce 2017, ch. I. In Bachelard, as in Enriques, the expressions *pensée des sciences* and *scientific thought* have a strong theoretical connotation, and they are intended to especially signify the intrinsic capacity of sciences to produce *tout court* thought along specific historical paths; it could be said without exaggeration that theirs are the only philosophies of science of the early 20th century which are concentrated systematically on the structure and history of scientific thought. At the same time it could be said that a greater awareness of their methods would allow us to combat with more appropriate weapons those philosophical positions, old and new, which, because anchored to the positivistic vision of sciences, negate their theoretical and cultural value in the sense of Poincaré.

[14] D. LECOURT, *La philosophie dans les sciences*, Revue de synthèse, (2005/2) t. 126, pp.451-454. And always following in the footsteps of Enriques and Bachelard, Lecourt directed the composition of the *Dictionnaire d'histoire et de philosophie des sciences,* P.U.F., Paris 1999.

[15] On the notion of epistemological profile, seen from the angle of the phenomenology of the

*phie du non* – conceived even before *Le rationalisme appliqué* (in which there appears a graphic that gives theoretical substance to the philosophical topology) – these representations are not the fruit of intellectual extravagance due to a lack of rigor or to literary diversions, as they appeared to some; rather, they are the results of precise methods which aim to grasp the internal dialectics of scientific thought and its fundamental concepts. And these dialectics are not reducible to either externally imposed schemas or to philosophies which exalt their results at the same time as being rendered already obsolete by the advance of the current research, which research, as a result of this advance, is in need of other, more complex approaches.

The primary Bachelardian aim, often not kept in mind adequately enough, is that of avoiding those "meta-scientific extensions" deriving from a theory which is seen as constantly *on the prowl*, an idea which had already been discussed in the first years of the 20th century by a figure like Maximilien Winter[16], still today completely unknown; this subtle epistemologist seems to almost use *ante litteram* a Bachelardian language and at the same time live in its identical atmosphere; and in fact he confronted the developments of the new logic and mathematical theories of the end of the 19th century, with the aim of creating an autonomous space for philosophical reflection, not to say a 'philosophical topology'. Some of his writings are oriented, just as are subsequently Bachelard's, towards understanding his contemporary science and addressing it with appropriate instruments, and it is no coincidence that they significantly appeared in the "Revue de Métaphysique et de Morale". In this journal, of which Winter was one of the founders, much space was dedicated from the very beginning to the critical study of the 'connaissances mathématiques', and to the point of branding it, as Alunni says, as a real journal of philosophy of mathematics, born with the precise aim of dealing above all with what was happening at the time within this science, and already with a decisively post-positivist perspective, as well as already oriented (like the almost concomitant first Vienna Circle) towards laying the foundations of a new approach in the field of the emerging epistemological literature, where such analysis, however, could not ignore the historical point of view[17].

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epistemic subject, see C. VINTI, *Il soggetto qualunque*, cit., chap. IX, and on the concept of the spectrum that finds its roots in the analysis of Hilbertian vectorial spaces, cfr. C. ALUNNI, *Spectres de Bachelard,* cit.

[16] M. WINTER, *La méthode dans la philosophie des mathématiques*, Alcan, Paris 1911, p. 51.

[17] C. Alunni in France has recently been engaged in the rediscovery of this figure, who had among other things a close relationship with Federigo Enriques. On this cfr. C. ALUNNI, *Maximilien Winter et Federigo Enriques: des harmonies exhumées*, in C. ALUNNI-Y. ANDRÉ, *Federigo Enriques o le armonie nascoste della cultura europea. Tra scienza e filosofia*, Edizioni della Normale, Pisa 2015, pp. 101-147 and *Federigo Enriques et la méthode histori-*

Thanks to the mediation of his master Léon Brunschvicg, the works of Bachelard are situated in this historico-theoretical juncture, little taken into account by scholars and even more radicalized in its different dynamics with the demand for a permanent semantic revolution at every level, even in the applied terminology; this demand finds its point of culmination precisely in *Le rationalisme appliqué* where that *rationalisme du mobile scientifique* is systematically put into action. Since language is non-neutral, being a depository of already consolidated points of view, it must be made 'mobile' to be able to fully grasp what is stirred up in scientific thought, even because whoever deals with the unique and unrepeatable moment of scientific discovery – a central theme for every philosophy of science – doesn't participate directly in that event and must reconstruct it through its texts. Well-known ideas such as epistemological rupture, epistemological obstacle and epistemological discontinuity will acquire therefore more theoretical sense if they are inserted in this context of total renovation of the categories of thought – a renovation which becomes, by extension, one that involves the very structures of the human 'esprit', the epistemic subject. The utilization, then, of prefixes such as 'sur', 'pan', 're', 'ex'[18] serves the project of restoring a qualitatively different meaning to the common terms of scientific and philosophic vocabulary because, as he often says in various works, every analysis is based on the texts of scientists whose different implicit and explicit historico-theoretical articulations must be brought out; the philosophy of science itself in its turn is called on to develop a role of total renovation of its own conceptual and linguistic structures, since its primary task is to bring back to life the difficult moment of the scientific discovery, l'*hinc* et il *nunc* of the genesis of new ideas which, as such, is not reducible to any prefixed schemas. The risk is to get caught up in certain theoretical accidents, due to the fact that such a philosophy could still have the necessity to simplify certain situation-types, when instead the scientific process remains structurally complex in every moment of its growth.

The recourse to philosophical topology serves the scope of eliminating underlying or implicit absolutist and normative positions, whether on the part of philosophers or scientists, and among other things it doesn't allow us to overlook Bachelard's interest in the corpus of mathematics, as it has been unfortunately done by some of his past critics. And even if such interest, as we have demonstrated in some of our other writings, is oriented towards comprehending the ever-growing processes of mathematization present in

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*co-critique*, in *Dossier Enriques*, "Kairos. Journal of Philosophy & Science", 14, (2015), pp. 75-116.

[18] For a more detailed analysis of the heuristic role of these prefixes in Bachelard's work, cfr. C. VINTI, *Gaston Bachelard, une épistémologie du sujet*, Mimesis, Milano 2014, cap. IV.

physics, to the point of becoming a point of reference for other French 20th century figures like first Albert Lautman and later Gilles Châtelet[19], it is always aimed at grasping the specific attributes of mathematics; their creative power through the process of ever-growing generalization and abstraction; their way of being always more rigorous even in the midst of continuous conceptual change, as Federigo Enriques had emphasized in *Per la storia della logica*[20]; their way of being and producing knowledge and of being, as a consequence, thought; their implicit philosophy. For this reason the typically French phrase *philosophie mathématique* is best suited to the Bachelardian path, because it is directed towards comprehending the 'synthetic' and at the same time historical character of the entire mathematical corpus. Such phrase suits Bachelard's work better than the term 'philosophy of mathematics' that, as it is known, belongs to another important line of epistemological thought.

The topology therefore can be considered as one of the different forms of translation in philosophical terms of the contents of knowledge produced by maths, which in their turn acquire more epistemic value in being further organized as mathematical physics, and in the consequent applicative implication; and if this outcome seems more evident in graphic representations, as in the profile and the spectrum, it is the *philosophie mathématique*, with its autonomous contents, which is the base of reference. At the same time the topology has the task of comprehending the scientific present, of grasping what is happening in the *tissu vivant* of mathematics, of catching what needs to be caught in its depth, outside of the existing philosophical framework; and even if it is the precipitate of a process begun in the first works, it also shows in a polemic manner the *synthetic* structural character of mathematics in a moment in which the analytic vision predominated[21]. The topology,

[19] But, as we have demonstrated, all this derives from Bachelard's epistemological deepening of the scientific work first of Reimann and later of Hermann Weyl, a fact not adequately emphasized by the scholars of his thought, with the exceptions of the work of Alunni and Palombi, already cited. Cfr. *Sur une petite phrase de Riemann*, cit. and *For an epistemology of mathematical contents: Albert Lautman*, "Lettera Matematica", International Editions Springer, 2018, pp. 1-10.

[20] Cfr. F. ENRIQUES, *Per la storia della logica*, Zanichelli, Bologna 1922.

[21] The 'synthetic' character of Bachelard's epistemology, or better said the insistence on the synthetic aspects of science present above all in *La Philosophie du non* and in the following works, has not been adequately analyzed. In fact, more than historical epistemology, we can better talk in his case of synthetic epistemology, both to distinguish it from the contemporary analytical epistemology and to grasp its own specificity. Here 'synthetic' refers to the fact that every theory, in the moment in which it is formed, brings in a new theoretical configuration and presents itself intentionally as a *transformative synthesis*, by making the *novum* interact with the past, which past is then redefined and at the same time delimited and made to become more rigorous in its own ambit. This is well expressed in *L'activité rationaliste de la physique contemporaine* and in the final chapter of *La Philosophie du non*, significantly titled

among other things, responds to this need for a faithful representation of the state of mathematical knowledge and its creative moments – with the overcoming of the stage linked to the logical-set-theory moment and a necessarily polemical charge in the background:

> And when one passes from science to the philosophy of science, the polemical aspect of the truth makes its appearance. This is so true that it could be said that the philosophy of science is that which in science belongs to polemical reason.[22]

For these reasons and to make clearer the Bachelardian path, it is not out of place to use the term 'topological reason' that comes to represent first and foremost that 'polemical reason' opposing the existing philosophies of science; at the same time such reason is able to present the possible spectrum of the implicit philosophies, which must nonetheless be tested on the ground of that applied 'dialogued philosophy' typical of the *nouvel esprit scientifique*, where the *raison mathématique* – a term in the French epistemological tradition used not by coincidence by Gödel[23] – plays, with its basic creative moments, a constitutive role in its diverse articulations[24]. Bachelard always held

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'La valeur synthétique de la philosophie du non'. In the mathematical corpus such 'synthesis' acquires particular connotations, as in the case of non-euclidean geometries, studied in *Le nouvel esprit scientifique*, tensorial calculation and the theory of groups, which also are some of Bachelard's objects of analysis. An organic synthetic philosophy of 'contemporary mathematics', in the Bachelardian sense of the term, and in the light of the results of Grothendieck, Serre, Connes, Attiyah, Kontsevitch, Zilber, Gromov and others, is present in the 2012 work of Fernando Zalamea, which develops some theses of Albert Lautman. Also cfr. F. ZALAMEA, *La philosophie synthétique des mathématiques*, tr. fr. by C. Alunni, Hermann, Paris 2018 and A. LAUTMAN, *La matematica come resistenza*, tr. it. and introduction by M. Castellana, Castelvecchi, Roma 2017, with an afterword by Zalamea.

[22] G. BACHELARD, *Le rationalisme appliqué*, cit., pp. 68-69. In the scientific path there has always been the need to sweep away pseudotruth through an engagement of reflexive character, as indeed said Federico Cesi who, in founding the *Accademia dei Lincei* in 1603, spoke of the necessity for those who were preparing to know the 'grand theatre' of nature to systematically carry out a 'philosophical *militia*'; see Carlo Vinti's introduction to his new critical edition of the works of F. CESI, *Il natural desiderio di sapere. The Natural Desire for Knowledge*, Pontificia Academia Scientiarum, Vatican City 2003.

[23] The term *raison mathématique* belongs to the Cartesian tradition and in France, together with the almost synonym *philosophie mathématique*, was resumed in the second half of the 19th century to discuss the new cognitive character assumed by mathematics after the advent of non-euclidean geometries and the work of Hermite; cfr. G. DARBOUX, *Éloges académiques et discours*, Hermann, Paris 1912, p. 142, a work used by K. Gödel to understand Hermite's particular 'platonist' point of view. Also cfr. K. GÖDEL, *Collected Works, Volume III: Unpublished Essays and Lectures*, Oxford University Press, New York, 1995, p. 323.

[24] Recently the work of Alexandre Grothendieck is receiving a certain critical attention aimed at explaining his unusual creativity in the field of the mathematical corpus; but a Bachelardian reading in the light of his 'synthetic' viewpoint could reveal itself to be very useful in trying

a non-static vision of mathematics, and saw in mathematics' structural changes not an illness, not the advent of the 'pathology of truth', to use a metaphor from the Bourbakist group, but the radical renewal of its mode of being with inevitable implications of theoretical character; and it is for this reason that he considers that which he calls "rapid philosophical topology" almost as "the keyboard on which tap most of the philosophical questions regarding science"[25]:

Idealism
↑
Conventionalism
↑
Formalism
↑
Applied Rationalism and Dialectic Materialism
↓
Positivism
↓
Empiricism
↓
Realism[26]

But to better comprehend the function of topological reason it would do well to keep in mind the entire Bachelardian path, which, beginning with *Essai sur la connaissance approchée,* was all oriented towards laying the foundations of a 'new epistemology', as it is clearly emphasized in *L'expérience de l'espace dans la physique contemporaine,* where an entire chapter is dedicated to the "philosophical consequences of the notion of mathematical operator". This 'new epistemology' of mathematics "must commit itself on two

to understand the inspiration, methodology, the creative processes put into being, his polemical force, his germinal and implicit epistemology and to avoid irrational interpretations of it. First attempts are present in the work of philosophers-mathematicians such as F. PATRAS, *La pensée mathématique contemporaine*, P.U.F., Paris 2001; ZALAMEA, *La philosophie synthétique* and R. GUITART, *Deux problèmes en vue d'une épistémologie transitive des mathématiques,* "Revue de synthèse", n.1-2, (2015), pp. 237-279 ; cfr. also AA.VV., *Matematica ribelle. Le due vite di Alexander Grothendieck,* Il Corriere della Sera, Milano 2014.

[25] G. BACHELARD, *Le Rationalisme appliqué*, cit., p. 7.

[26] Ivi, p. 5. For a more detailed analysis of the topology and of the Bachelardian critiques to formalist positions, cfr. F. PALOMBI, *Elogio dell'astrazione*, cit., pp. 81-87. But Lecourt had already stressed the role of the arrows, "symmetrical and opposed to the central line", and at the same time highlighted the way in which the topology analyzes "the nature of each of the doctrines which resides not in them [the arrows] but in the fold as a fixed element of philosophical space". Cfr. D. LECOURT, *L'epistemologia storica di Gaston Bachelard*, in G. CANGUILHEM-D.LECOURT, *L'epistemologia di Gaston Bachelard*, ed. by F. Bonicalzi, Jaca Book, Milano 1997, p. 129.

opposite fronts: against the followers of mathematical symbolism, and against the followers of philosophical realism". Against the supporters of these two point of views, and against others that under different forms are only their uncritical offshoots, "we give the same response: mathematics, with its inventive thought, goes beyond both conventions and experience"[27]. Thus the topology presents itself as a theoretical device, as a real conceptual technique for critically reviewing – both from an historical and above all from a properly theoretical point of view – the epistemic sense of mathematical theories in their almost necessary articulation towards physical-mathematical conceptual organizations. All this is done in the wake of the work of Reimann, Weyl and of a certain Hilbert, with the aim of overcoming those naïve visions of Platonism present in various ways in the reflections both of scientists and of philosophers of mathematics[28]. At the same time the

[27] G. BACHELARD, *L'esperienza dello spazio nella fisica contemporanea*, tr. it. by M.R. Abramo, A. Siciliano Ed., Messina 2002, p. 56 and p. 58; This work, like the second chapter of the 1929 *La valeur inductive de la relativité*, reaffirms 'the inductive value of mathematics' always on the bases of the analyses of the philosophical consequences of the advent of tensorial calculation, considered decisive in the relativistic doctrines. Charles Alunni has emphasized the particular 'inductive value' attributed to mathematics by Bachelard and the constitutive role, as well as 'synthetic', which that assumes in his theoretical device. Again, a little studied topic. See cfr. C. ALUNNI, *Spectres de Bachelard*, cit.

[28] To understand Bachelard's position towards Hilbert, studied first in *L'expérience de l'espace dans la physique contemporaine* of 1937 and then in *Le Rationalisme appliqué*, it is also important to take account of Bachelard's reading of the first works of Albert Lautman, appeared first in 1938, works in which much space is dedicated to the works of the German mathematician and to a way of reading their philosophical implications which goes beyond the interpretations given at the time by members of the Circle of Vienna. See *Essai sur les notions de structure et de existence en mathématiques* and *Essai sur l'unité des sciences mathématiques dans leur développement actuel* (Ed.Vrin, Paris 2006); if the second Bachelard in the epistemological field (1949-1953) is more oriented towards a synthetic vision of his philosophy of science, perhaps this is due to the work of this young philosopher of mathematics and not coincidentally in a letter sent to him in 1938 Bachelard writes that he is "bien frappé par la richesse des aperçus" and above all for having "su garder la pensée, en évinçant les calculs, difficulté Presque insurmontable"; see cfr. H. BENIS-SINACEUR, *Lettre inédite de Gaston Bachelard à Albert Lautman*, "Revue d'histoire des sciences", t. XL, 1, (1987), p. 129. These *aperçus,* emphasised by Bachelard, are the fruit of an unusual reading by Lautman of the mathematical theory of the early 20th century, and of prime attention given to the theorems of Gödel and Gentzen, who was seen in a philosophical position considered unfitting with the philosophies of mathematics of the time, as indeed the Swiss philosopher of mathematics Ferdinand Gonseth was warning at the end of the '30s, the author of works such as *Les fondements des mathématiques, de la relativité générale à l'intuitionnisme* (1929), *Les Mathématiques et la réalité mathématique* (1936), *Philosophie mathématique* (1939). A first exposition of the need for a new philosophy of mathematics and of a 'new epistemology', different from that which was about to become the *Standard* current, was put forward at the second Congrès de Philosophie Scientifique in Paris in 1937 by Gonseth, Cavaillès and Lautman for mathematics and by Enriques for philosophy of science. On this cfr. *Alle origini della 'nuova epistemologia'. Il Congrès Descartes del 1937*, cit. and A. Lautman, *La ma-*

topology has the function of sweeping away from philosophical analysis the consideration of mathematics as a language, consideration which the topology sees as a commonplace accepted almost as an inherent fact of mathematics' nature. Such a consideration is made with diverse motivations and variables by different strands of epistemological thought which are inattentive to their own particular historical and 'synthetic' nature:

> It is necessary to break with that cliché which is dear to certain sceptic philosophers that want to see in mathematics only a *language*. On the contrary, mathematics is a *thought*, a thought secure in its own language. The physicist think experience with this mathematical thought… To comment on experimental results without giving an account of theoretical and mathematical preparations would be like putting to one side some of the greatest lessons of synthetic philosophy… But the axis of the experimental determinations of contemporary science doesn't follow the directions postulated by Platonic Realism… One must have an awareness of abstraction in the moment in which some abstract plans are needed to inform experience[29].

The Bachelardian topology or *raison topologique*, strengthened by this synthetic vision of mathematics considered implicit in the corpus and conceived as thought, assigns the correct role to that process which leads to the "formation of formalism", which process reinforces itself thanks to the "results of rational thought, but which cannot exhaust all of the work of rational thought"; at the same time the topology serves to eliminate as much as possible another commonplace typical of the philosophies which consider mathematics to be a language, which see "in theoretical science a collection of *conventions*, a series of thoughts more or less comfortably organised in the clear language of mathematics, which is nothing but the *esperanto* of reason."[30] But this long battle against the consideration of mathematics as the 'esperanto of reason' finds its justification also in the *philosophie du mobile scientifique* and above all in the *enjeux*[31] that distinguish that; it methodolog-

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*tematica come resistenza*, cit. To such an end it is to be kept in mind that Bachelard maintained a constant rapport with Ferdinand Gonseth, to the point of being the cofounder in 1947 of the magazine 'Dialectica' together with Paul Bernays; for a comparison between Bachelard and Gonseth, cfr. C. VINTI, *Présence de Ferdinand Gonseth dans la pensée de Gaston Bachelard*, "Revue de synthèse", n. 2, (2005), pp. 391-415.

[29] G. BACHELARD, *L'activité rationaliste de la physique contemporaine*, cit., pp. 29-30.

[30] G. BACHELARD, *Le Rationalisme appliqué*, cit., p. 5.

[31] The term *enjeux* is often found in the works of Bachelard, but it was Gilles Châtelet afterwards who gave it a more precise meaning in studying the work of Grassmann, Argand, Maxwell, Hamilton. cfr. G. CHÂTELET, *Les enjeux du mobile. Mathématique, physique, philosophie*, Ed. du Seuil, Paris 1993 ; on Châtelet cfr. *Razionalismi senza dogmi*, cit., chap. VI; C. ALUNNI, *Des 'Enjeux du mobile' à 'L'Enchantement du virtuel' - et retour*, Introduction to G. CHATELET, *L'enchantement du virtuel. Mathématique, physique, philosophie*, by C. Alunni

ically carries out the scientific *liquidation du passé*, but to insert it in a more general context, where it acquires more sense to the extent that its basic concepts change meaning, as is affirmed in other works; and at the same time it nails down the philosophies and the philosophic space to their time, revealing in them their delayed structures compared to the real advance of evermore complex knowledge, in need of qualitatively different approaches with the contextual criticism of the fundamental traditions. At the same time, the introduction of the philosophical topology has the aim of "asserting some epistemological *nuances* that a more blocked, static thought can obliterate"[32]. Such epistemological *nuances*, even if sometimes they are not visible, are present in the *tissu* and the folds of science, as Ludovico Geymonat will say; furthermore, as Moritz Schlick[33] had already remarked with other words in 1918, the philosopher of science has the obligation of bringing these nuances to the light, thus giving voice to the philosophical spirit of science.

The Bachelardian topology, even if "it was established paying specific attention to the physics and chemistry contemporary to him, seems to possess a more general value"[34]; it is its open and problematic character that lends it this potential to be used in other contexts, a character developed due to the fact that it deals with the *enjeux* and the risks, whether epistemological or ontological, that are produced by science in general and mathematics in particular in their history starting from non-euclidian geometry – an object of constant analysis to the point of becoming the mark of a way of understanding the work of philosophy as the 'philosophy of the non'. For this reason it can be considered heuristically useful first of all to reformulate it by inter-

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and C. Paoletti, Ed. Rue d'Ulm, Paris 2010, pp. 9-56 and the recent number of the "Revue de synthèse", n. 1-4, (2017), *Philosophie contemporaine des matématiciens: Évariste Galois, Gian-Carlo Rota, Gilles Châtelet*, by C. Alunni, Y. André & C. Paoletti.

[32] G. BACHELARD, *L'esperienza dello spazio nella fisica contemporanea*, cit., p. 58; the Bachelardian insistence on the term *expérience* present in various works must be emphasised. In fact it is common in French epistemology, for example in the work of Cavaillès and Lautman, and is an expression of their idea of the philosophical analysis of concepts as being direct participation in the *veçù* of science and in, as Federigo Enriques said, the 'labour of concepts', attentive to their creative and inventive moments that avoid getting stuck in a 'blocked thought' and then producing 'metaextensions' of a theory.

[33] M. SCHLICK, *Teoria generale della conoscenza*, trad. it., F. Angeli Ed., Milano 1986, p. 12. A critical history of the philosophy of science among other things demonstrates that in their beginnings Schlick and Bachelard had many objectives in common with almost identical propositions. For a first attempt to study Schlick and Bachelard together, cfr. *Spazio e tempo nell'epistemologia di Gaston Bachelard*, in *Gaston Bachelard. Bilancio critico di una epistemologia*, by M. Castellana, "Il Protagora", 5, anno XXIV, (1984), pp. 27-43. In this monothematic issue of the magazine 'Il Protagora' there are essays by G. Canguilhem, M. Loi, R. Cavaillès, V. Tonini, M. Quaranta, C. Vinti and others.

[34] F. PALOMBI, *Elogio dell'astrazione*, cit., p. 83.

vening on some gaps, as Fabrizio Palombi did, integrating it from above with the 'Phenomenology' and from below with the 'Intuitionism', and at the centre with the 'Phenomenotechnique', to give account to the Bachelardian wake of that "polyphilosophy of mathematics"[35]. After all, Bachelard himself invites us on the first page of *Le rationalisme appliqué* to "widen a little this philosophical spectrum without modifying the order of the fundamental philosophies", always with the objective of being able to grasp some of the "new philosophical *nuances*"[36].

But it must be kept in mind that, based on the Bachelardian topology, or better, on topological reason, the analytic/synthetic dyad (or poles, to use the Bachelardian language) comes to play a decisive role. These two poles, even if it is difficult to give them the right collocation inasmuch as they are more than philosophical positions, represent two essential moments of mathematical operations, which are not easily comprehensible if not from within a viewpoint more attentive to those operations' particular historical dimension. Despite the asymmetry between the two, they give life to mathematics' always-open living *tissu*, where that specific object of the *philosophie mathématique*[37], the continuous dialectic between *schémas de structure* and *schémas de génèse* (to use the terms of Albert Lautman), is realized. Indeed, thanks to the continuous encounter of these two ways of understanding and practising mathematics, mathematical creativity is enhanced, and for this they can be considered some of the invariant factors that permeate the historical-conceptual path, rich with articulations and ever-new philosophical *nuances*. Having kept in mind this internal dialectic of mathematical thinking has allowed the Bachelardian path to historicize the idea of 'scientific philosophy' and to see in it various levels corresponding to the different stages of the *raison mathématique*, from Euclid onwards. The proposed philosophical topology fully satisfies these continuous demands for 'new apodicticity', for transformation and deformation of the base concepts that the traditional theory of knowledge was not able to introduce into its epistemic circuit, inasmuch as it was tied to concepts believed to be unchangeable.

We want to make ours some of the solicitations present in *Le rationalisme appliqué* to try to understand the concomitance of certain philosophical *nuances* present in other almost contemporaneous paths, like that of Kurt Gödel, and also to avoid defining him, as has been done recently, as "the great Prince of Darkness of modern mathematics", because with its theorems "it finds itself suspended in mid-air"[38], lacking in foundations because it is

[35]Ibid., p. 84-85.
[36] G. BACHELARD, *Le Rationalisme appliqué*, cit., p. 7.
[37] Cf. A. LAUTMAN, *Essai sur les notions de structure et d'existence en mathématiques*, cit.
[38] D.F. WALLACE, *Tutto, e di più. Una storia compatta dell'infinito*, (2003), trad. it., Codice,

not easy to accept the idea, as Bachelard pointed out, that it is possible to construct without foundations. Even a cursory study of these two philosophers bears witness to the fact that from the comparative analysis of determinate problems, from the "exchange of protocols of a problematic", "an atom of rational communion" can appear, a form of "interrationalism", "the union of the proof's workers". Faithful to this Bachelardian appeal – to understand what was slowly emerging in the *philosophie mathématique* of the 1950's, starting from the "understanding of the stating of a problem" – one can also "develop a type of topology of the problematic"[39], that is the desire and at the same time the need for a different viewpoint with which to look at what was occurring in that which Hermann Weyl called the vast "granite empire of mathematics", subjected as it was to profound transformations of its base concepts with the creation of new sectors and the emergence of new questions. In the light of this extension of the idea of 'topology of the problematic', or topological reason, the entire path of Austrian logic can be seen as a constant and successful attempt to construct "the edifice of mathematics" without foundations and to theoretically live in such an atmosphere; in this way the considerations present in various of Gödel's essays of the 50's and 60's can acquire more epistemic meaning, essays in which, to use Bachelardian language, can be glimpsed an implicit philosophy, or better a germinal epistemology, sometimes more sketchy and sometimes more hidden: *Some basic theorems on the foundations of mathematics and their implications* (1951), *Is mathematics syntax of language?* (1953/59), *The modern development of the foundations of mathematics in the light of philosophy* (1961/?)[40].

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Torino 2005, pp. 228 note and 235 note.

[39] G. BACHELARD, *Le rationalisme appliqué*, cit., p. 56.

[40] K. GÖDEL, *Collected Works,* cit., pp. 304-323; pp. 335-362 and pp. 374-386. As is known, there are different versions of some of these unpublished essays, in particular those between 1953-59; this indicates one the one hand the scrupulousness of Austrian logic and on the other the difficulty of putting forward a more organic philosophical point of view. However these essays are the testimony to the sufferings of a philosophical 'pilgrimage', and must be taken in this sense not only for what they have effectively produced, but also as indices of a path of which we can gather indications of a certain significance, as indeed has been done *in primis* by Hao Wang – who as is known had various meetings with Gödel – and also by Jean Petitot in the '90s who developed the idea of a transcendental Platonism. Other than the curators of the *Collected Works*, there are some recent scholars to which we refer who have tried to put them forward in a more organic way; among these cfr. F. RODRIGUEZ-CONSUEGRA, *Un inédito de Gödel contra el convencionalismo*: *historia, anàlisis, contexto y traduction*, "Arbor", CXLII, (1992), pp. 323-348) and *Kurt Gödel. Unpublished Philosophical Essays*, Birkäuser Verlag, Basel-Boston-Berlin 1995; J. PETITOT, *Pour un platonisme transcendantale*, in M. PANZA- J.M. SALANSKIS (eds.), *L'objectivité mathématique. Platonisme et structures formelles*, Masson, Paris 1995, pp. 147-172; G. LOLLI, *Incompletezza. Saggio su Kurt Gödel*, Il Mulino, Bologna 1992; *Da Euclide a Gödel*, Il Mulino, Bologna 2004 and *Sotto il segno di*

The Bachelardian reading of these essays of Gödel is compels us not to see the two men separately; rather to see them together as a sign of a real strategy aimed at delineating a vision of mathematics which does not easily fit with those other visions dominant in that period, a vision distant from the "prejudices in vogue", as Gödel himself often says, for example in letters to Wang. At the same time the Bachelardian reading can help to review in another light Gödel's interest in Kantian philosophy[41] and also in the philosophy of Leibniz, as well as, in a more limited but not random way, in the philosophy of Husserl, so as to also insert these figures in a more general context, the way it came to shape itself in other concomitant paths.[42] In fact, a critical history of the philosophy of science can't fail to give these two the right importance and at least allows us to consider them some signals of some other cultural atmosphere that is struggling to emerge. At the same time, attentive to the real changes taking place, it helps us to situate the 'topology of the epistemological problems' in a more appropriate context, and again allows us to see more 'atoms of rational communion', more 'proof's workers' engaged in an 'interrationalist' path, the forming of a small epistemic community that, while being small, feels the need to put forward another viewpoint with which to consider the Weylian 'empire of mathematics' if not the idea of a further 'breakthrough', in the Schlickian sense of the term, in the epistemological literature.

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*Gödel*, Il Mulino, Bologna 2007; J. HINTIKKA, *On Gödel*, Wadsworth, Belmont 2000; M. DAVIS, *What Did Gödel Believe and When Did He Believe it?,"*The Bulletin of Symbolic Logic", vol. 11, n. 2, (2005), pp. 194-206; D.A. MARTIN, *Gödel's Conceptual Realism*, "The Symbolic Logic", vol. 11, n. 2, (2205), pp. 207-224; D. CHIFFI, *Kurt Gödel, Philosophical explorations*, Aracne Ed., Roma 2012 and R. BRUNI, *Kurt Gödel, un profilo*, Carocci Ed., Roma 2015. A separate mention can be made for P. CASSOU-NOGUÈS, *I demoni di Gödel. Logica e follia*, trad. it., B. Mondadori, Milano 2008, which illustrates what is called 'fantastic metaphysics', the relationship between 'madness and logic' which helps to understand the various obsessions of Gödel, who was fascinated by esoteric questions.

[41] Cf. also the essays on the relationship between relativity and Kantian philosophy at the end of the 1940s (ibid., 247-259), that give an idea of the Kantian origins of his thinking, as is indeed confirmed in a letter written in response to a questionnaire sent to him by a PhD sociology student: "only Kant has had any influence on my philosophical thought in general". See cfr. K. GÖDEL, *Collected Works*, *Volume IV: Selected Correspondance A-G*, Oxford University Press, Oxford 2013, p. 297.

[42] The figures of Suzanne Bachelard and Jean Desanti must be kept in mind at the end of the 1950s, at least for France. They engaged themselves with Husserlian thought to find a different path from the so-called *Standard* philosophy of science, they were reaching for a 'phenomenological epistemology' of mathematics; cf. S. BACHELARD, *La conscience de rationalité*, P.U.F., Paris 1958 and J. DESANTI, *Les idéalités mathématiques*, Le Seuil, Paris 1968 and on these two figures cfr. *Razionalismi senza dogmi*, chap. V e *Epistemologia debole*, Bertani, Verona 1986, chap. II.

It could be said that Gödel practised from the beginning of his career that approach which Bachelard called in *La Philosophie du non* 'the philosophy of why not?', that is the need for science in general to go beyond already established knowledge, to think of alternative possibilities through the metamathematical gaze that lies outside of any specific sector, that which later Gregory Chaitin will define as "a wonderful metamathematical result. Metamathematical because it is not a mathematic result, rather it is a theorum about mathematics itself, about the limits of mathematic methods"[43]. In this way the critical point of view was used heuristically as a precondition of the theorum of incompleteness, through the awareness of the non-exhaustive value of Hilbert's hypothesis on the completeness and the demonstrability of formal systems. All this allowed Gödel to dialecticize in Bachelardian terms the available material of the work of logic, that is to vary the syntactic scope of the liars paradox, to problematize it and to put into practice *les Méthodes nouvelles*, about which Henri Poincaré spoke, that lead to negative results. In saying 'no' in the Bachelardian sense one arrives systematically at setting some limits and revealing the instruments needed to reactivate on new bases the procedures and the techniques that lead to the establishment of a new field, more delimited and at the same time more precise. Gödel made of the reflexive moment an indispensable instrument for thinking of a different form of the construction of objectivity; to use an expression of Yehuda Elkana, it can be said that he accomplished a "scientific revolution as a revolution in reflexivity"[44]. This awareness becomes more apparent in his essays of the 50's and 60's, and it almost brings him to perceive in his theorems an 'epistemological break', inasmuch as they are carriers of a *nouvel esprit scientifique*, even if he sometimes makes these implications appear only like some *nuances*, because he believes, as Bachelard himself on many occasions had emphasised[45], that the philosophical reflexion on these theorems is structurally delayed.

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[43] G. CHAITIN, *Alla ricerca di omega*, (2005), trad. it., Adelphi, Milano 2007, p. 34.

[44] Cf. Y. ELKANA, *La rivoluzione scientifica come rivoluzione nella scientificità*, in A. LA VERGATA-A. PAGNINI, *Storia della filosofia. Storia della scienza. Saggi in onore di Paolo Rossi*, La Nuova Italia, Firenze 1995, pp. 23-35.

[45] This also explains his difficulties in publishing these writings, in subjecting them to continuous verification, and similarly the fact of refusing to participate for example in a more philosophical relationship with the *Entretiens de Zürich sur les fondements et la méthode des sciences mathématiques,* organised in 1938 by Ferdinand Gonseth who, through Paul Bernays, had officially invited him, as is clear from Gödel's letter of sent to Gonseth on the 7th November to thank him for the invite, where among other things Gödel communicates being about to conclude the demonstration of Cantor's continuum hypothesis. (We have the Director of Losanna Library, where the Gonseth fund has still been little explored, to thank for having delivered the letter). This *Entretien* was formed in the December of 1938 with the objective of beginning to think about a different model to that *Standard* of the philosophy of mathematics,

In the *Nachlass* and in various letters of late Gödel, and in that which is called his 'philosophical stage'[46], we can find this need that translates into a real path, even though it might not be linear but instead subject to and riddled with various changes of mind. This path is aimed at laying the foundation for a difference philosophical image of mathematics, not reducible to the canon of those images current at that time. Gödel was conscious of some new *nuances* implicit in his results, but he was also conscious that these *nuances* were in need of a different theoretical platform to be able to be correctly understood. One *nuance* that, thanks to his direct philosophical commitment, he tried hard to clarify, and that he felt would not be well metabolised by the philosophic-scientific environment, was the idea, of which his theorems were the expression, that the truth of mathematical utterances and axioms did not coincide with their demonstrability. At the same time this required an epistemological attitude that in Bachelardian terms can be defined as *mobile*, in virtue of the fact that their contents by nature are always on the point of being an object of continual revision. Hence comes the epistemic realization of the insufficiency of those 'fashionable' positions that considered exclusively their stability and aimed at the absolute self-foundation of the system of knowledge, and the warning that in fact those positions could even constitute some 'epistemological obstacles' for an adequate comprehension of the contents of mathematics because the problem concerning the nature of mathematics, considered crucial by Gödel, was not seen in its proper dimension.

A Bachelardian reading of these essays first and foremost helps us to better understand Gödel's efforts aimed at claiming for the mathematical corpus an objectivity not reducible only to the formal-constructivist aspect, as well as his continuous insistence on the 'highly transfinite concept of objective mathematical truth', derived from the results of incompleteness and undefinability. At the same time a Bachelardian reading allows us to see in a different light the constant critiques to conventionalist visions contained in these Gödel's essays, which, not by chance and in many places, in a way similar to Bachelard's, emphasize the particular character of mathematics: its producing 'contents', its not being a 'language', as it is repeated and highlighted with diverse arguments in the six versions of his tormented essay *Is*

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as it was emerging on the basis of some results among which were Gödel's, who was invited precisely for this reason. Among the participants were Bernays, F. Enriques, M. Frechet, J. Jorgensen, H. Lebesgue, J. Lukasiewicz, G. Polya, Th. Skolem and others such as Ackerman, Brouwer, Gentzen, Heyting and von Neumann who for various reasons could not participate. cf. F. GONSETH, *Les Entretiens* de *Zürich sur les fondements et la méthode des sciences mathématiques*, A. Leeman, Zürich 1941.

[46] F. RODRIGUEZ-CONSUEGRA, *Un inédito de Gödel contra el convencionalismo*: *historia, anàlisis, contexto y traduction*, cit.

*mathematics syntax of language*? All this leads to the powerful idea that mathematics produces true knowledge, always *approchées* and *inachevées*, which is incomplete; at the same time the idea emerges that there exists a close analogy between mathematics and theoretical physics, which formalist visions, considered in Winter's terms as undue 'metaextensions' of a determined scientific doctrine of the time, did not emphasize enough.[47]

Already Albert Lautman in the late '30s, thanks to his critical analysis of the theorems of incompleteness, warned of the fact that the philosophies of mathematics of that time were very insufficient, and that they offered a spectacle, to say the least, "*décevant*"; he went so far as to delineate the need for a real 'resistance'[48] against that which Gödel defined as 'nominalistic philosophies of mathematics' of a positivistic derivation in their different variables, starting with the contemporary, conventionalist positions.

These results, far from being considered, as Jean-Yves Girard would say, "the anti-scientist panacea"[49], pose a fundamental question concerning the nature of mathematical knowledge, in highlighting the necessity to go beyond the formal-syntactic vision; furthermore, they introduce the idea of a particular role being given to the intuition which assumes, to use another Bachelardian term, the physiognomy of an *intuition travaillée et mobile* in its allowing access to mathematical objectivity and in its bringing to light the non-autonomy of logic; this because in order to be productive logic must be engaged, as will be said with rare theoretical sensitivity by Hélène Metzger in her original studies of the history of science[50].The non-autonomy of logic

[47] We can say that in these essays of Gödel there breathes a Bachelardian air, almost familiar, with points of view and thematics which make an integral part of every discourse of every authentic philosophy of mathematics and of science in general. As Vincent Bontems says, even if in Bachelard there is not a single reference to Gödel and his theorems, because already from his first writings that emerged in the 1930s there was a severe criticism of all those attempts which tended towards the absolute auto-foundation of a system of knowledge, his vision, as far as it concerns logic, can be interpreted in the light of the successive works of Paul Cohen on *forcing.* cf. V. Bontems, *À la pointe du rationalisme*. Introduction to *Gaston Bachelard et l'avenir de la culture*, cit.

[48] A. LAUTMAN, *La matematica come resistenza*, cit.

[49] J.Y. GIRARD*, Le Point aveugle. Cours de logique. Vers la perfection*, Hermann, Paris 2006, p. 30; in the first part of this volume we can find discussed the logical-philosophical sense of the idea of incompleteness. Linear logic, proposed in the '80s by Girard, could be considered a development of some points of the Gödelian path, starting from the critique of that which is called 'the formalist ideology' and from putting at the centre the question of the non-autonomy of logic. It must be emphasised that in the French world the question of the non-autonomy of logic has been at the centre of constant reflexion and heated debate starting from the foundation of the 'Revue de Métaphysique et de Morale'.

[50] Cf. H. METZGER, *Il metodo filosofico nella storia delle scienze,* in F. ENRIQUES-H. METZGER, *Storia e struttura del pensiero scientifico*, tr. it. by M. Castellana, Barbieri-Selvaggi Ed., Manduria 2014, p. 69.

is another culmination of the heuristic use of Gödel's methodology focused on "negative results", but it requires a not inconsiderable reflexive effort to be adequately understood as a real 'epistemological rupture', first and foremost in combating the static and conventionalist visions of mathematical thought, grafted onto which logic finds its reason for being and the productivity of its important procedures.

Within such a context, or 'topology of the problematic', Gödel's desire – to classify the philosophies of mathematics in order to reform the very same mathematics starting from its own concepts – acquires greater meaning. One among these concepts is for example the 'set'. In the 1961 essay *The modern development of the foundations of mathematics in the light of philosophy* he describes, in the words of the curators of his *Nachlass*, "in philosophical terms the development of the study of the foundations of mathematics in our century and fit it into a general scheme of possible philosophical *Weltanschauungen*"[51].

Analogically to the Bachelardian philosophical topology, even if in a horizontal sense rather than a vertical, Gödel offers us a double topology: that of the "possible conceptions of the world" and that more relative to the different philosophies. One could attempt therefore to topologize first that which he calls the "general scheme" that includes "two groups", one on the left and the other on the right. On both sides there are those which could be called large conceptual containers of visions of the world on the basis "of their affinity toor, respectively, turning away from metaphysics (or religion)"[52]; this scale could therefore be arranged in a horizontally rather than vertically.

[spiritualism] [idealism] [theology] [skepticism] [materialism] [positivism]

Such a scheme is considered useful heuristically and at the same time "fruitful for the analysis of philosophical doctrines" that have as their specific object the theory of knowledge; but Gödel is not content with this first 'scheme' and proposes another to complete it: "Thus one would, say that apriorism belongs in principle on the right and empiricism on the left sides", and at the same time "optimism belongs in principles toward the right and pessimism toward the left"[53]. The scheme can therefore continue to be represented in a horizontal fashion: mathematics goes on the right, because by nature it is considered an *a priori* science.

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[51] Cf. D. Føllesdal, in K. GÖDEL, *Collected Works,* Volume 3, cit., p. 364; refer to this *Nota* for the analysis and comment on his thought.

[52] K. GÖDEL, *The modern development of the foundations of mathematics in the light of philosophy*, p. 375.

[53] Ibidem, p. 377.

[apriorism] [optimism] [empiricism] [pessimism]

But starting from the philosophy of the Renaissance another *Zeitgeist* came to oppose such an aprioristic vision of the right, a *Zeitgeist* which pointed out the empiric aspect of mathematics. This has resulted in a notable development of its abstract capacities and of studies concerning its fundamentals. But in the late 19th century a series of new developments, like the antinomies of set theory, made their appearance, causing the aprioristic position to slip towards empiricist positions to the point of denying that "mathematics, as it had developed previously, represents a system of truths." Faced with "these nihilistic consequences" that find their reasoning in the "spirit of the time", in the mathematical field there was a reaction that was concretized in the project of Hilbert, which is "that curious hermaphroditic thing that Hilbert's formalism represents, which sought to do justice both the spirit of the time and to the nature of mathematics."[54] But Gödel considered this Hilbertian mediation impossible because the problems posed by the two traditional positions of the left and right lead to inevitable overlapping and so to evident contradictions: "one must either give up the old rightward aspects of mathematics or attempt to uphold them in contradiction to the spirit of the time. Obvious the first course is the only one that suits our time and [is] therefore also the one usually adopted. One should, however, keep in mind that this is a purely negative attitude."[55]

For this reason Gödel deems it necessary to embark on a new way, keeping in mind the "advances… are due just to this left-ward spirit in philosophy and world-view", progress in part due to the "excesses" and the "wrong direction of the preceding rightward philosophy". While preferring the right, as has been underlined by many, Gödel considers it necessary to find a median "the correct attitude appears to me to be that the truth lies in the middle or consists of a combination of the two conceptions"[56]. Bachelard in his topology had indicated in the 'applied Rationalism' and in the 'rational Materialism' his median or dialogued way of evaluating the epistemic value of philosophical theories and their degree of distance from the *nouvel esprit scientifique*. Similarly, Gödel, demonstrating himself to be always faithful to the heuristic principle of the objective mathematical truth, advances a more concrete proposal that passes from the real contents of a theory to identifying the underlying problems that the work of mathematics entails. Instead of founding the certainty of mathematics, it is necessary to construct it, as Bachelard said. Then, one can "by cultivating (deepening) knowledge of the

[54] Ibid., p. 379.
[55] Ibid., p. 381.
[56] Ibidem.

abstract systems, and themselves which to the setting up of these mechanical systems, and fur their by seeking, according to the same procedures, to gain insights into the can solvability, and the methods for the solution, of all meaningful mathematical problems… The procedure must thus consist, at least to a large extent, in a clarification of meaning that does not consist in giving definitions"[57].

If Bachelard and Gödel both came to propose two respective and original philosophical topologies, this is the result of the need to delineate the outlines of a philosophical strategy able to bring out that emerging *novum* in the complexity of scientific research which the theoretical methods of their time were not able to grasp. But as Hermann Weyl said in *Raum-Zeit-Materie* in 1918, "the first thing to do is to comprehend this fact: despite the oscillations of philosophy from system to system and all its slips, we must not give up this quest, because otherwise knowledge will become incomprehensible chaos."[58] The two proposed topologies fully respond to this need and as such must be reconsidered because, perhaps still today, they have something to tell us, we who live and act "in the ungrateful land of the philosophy of science", as Jean Cavaillès defined it in a letter to his friend Albert Lautman[59].

## REFERENCES

Aa.Vv., 2014, *Matematica ribelle. Le due vite di Alexandre Grothendieck*, Il Corriere della Sera, Milano.

Alunni, C., 1999, "Relativité et puissance spectrales chez Gaston Bachelard", "*Revue de synthèse*", n. 1: 73-110.

Alunni. C., (cur.), 2005, *Science et philosophie au XX^e siècle. L'École de Zurich et le programme surrationaliste,* "*Revue de synthèse*", t. 126.

Alunni, C., 2010, *Des 'Enjeux du mobile' à L'Enchantement du virtuel' – et retour*, Introduction to Châtelet G.,, *Les enjeux du mobile. Mathématique, physique, philosophie*, Le Seuil, Paris: 9-56.

Alunni C., Castellana M., Ria D. et Rossi A. (cur.), 2010, *Albert Einstein et Hermann Weyl 1955-2005. Questions épistémologiques ouvertes,* Barbieri-Selvaggi Ed.- Rue d'Ulm Ed., Manduria.

Alunni, C., (cur.) 2015, *Philosophie et mathématique. Bachelard et les mathématiques*, "*Revue de synthèse*", t. 136.

Alunni, C., 2015, "Maximilien Winter et Federigo Enriques: des harmonies exhu-

[57] Ibid., pp. 383.

[58] H. WEYL, *Temps, espace, matière*, tr. French, Blanchard, Paris 1922, p. 8.

[59] H. BENIS-SINACEUR, *Lettres inédites de Jean Cavaillès à Albert Lautman*, "Revue d'histoire des sciences", cit., p. 117.

mes", in Alunni, C.-André, Y, (cur.), *Federigo Enriques o le armonie nascoste della cultura europea. Tra scienza e filosofia*, Ed. Della Normale, Pisa: 101-147.

Alunni, C., 2018, *Spectres de Bachelard. Gaston Bachelard & l'École surrationaliste*, Hermann, Paris.

Bachelard, G., 1951, *L'activité rationaliste de la physique contemporaine*, P.U.F. Paris.

Bachelard, G., 1970[4], *Le rationalisme appliqué*, P.U.F., Paris.

Bachelard, G.,1970[5], *La philosophie du non*, P.U.F., Paris.

Bachelard, G., 2002, *L'esperienza dello spazio nella fisica contemporanea*, tr. it. by M.R. Abramo, Siciliano Ed., Messina.

Bachelard, S., 1958, *La conscience de rationalité*, P.U.F. Paris.

Benis-Sinaceur, H., 1987, "Lettre inédite de Gaston Bachelard à Albert Lautman", "*Revue d'histoire des sciences*", t. XL: 129.

Bontems, V., (cur.), 2018, *Bachelard et l'avenir de la culture. Du surrationalisme à la raison créative,* Presses des Mines, Paris.

Bruni, R., 2015, *Kurt Gödel, un profile*, Carocci Ed., Roma.

Canguilhem, G., 1992, *Ideologia e razionalità nella storia delle scienze*, tr. it, La Nuova Italia, Firenze.

Cassou-Noguès, P., 2008, *I demoni di Gödel. Logica e follia*, tr.it., Mondadori, Milano.

Castellana, M., 1974, *Il surrazionalismo di Gaston Bachelard*, Ed. Glaux, Napoli.

Castellana, M., 1984, "Spazio e tempo nell'epistemologia di Gaston Bachelard", "*Il Protagora*", n. 5: 27-43.

Castellana, M., 1986, *Epistemologia debole. Bachelard, Desanti, Reymond*, Bertani Ed., Verona.

Castellana, M., 1992, *Alle origini della 'nuova epistemologia'. Il Congrès Descartes del 1937*, Il Protagora, Lecce.

Castellana, M., 2004, *Razionalismi senza dogmi. Per una epistemologia della fisica matematica*, Rubbettino Ed., Soveria Mannelli.

Castellana, M., 2007, "Il metodo storico in filosofia della scienza", in Enriques, F., *Il significato della storia del pensiero scientifico*, Barbieri-Selvaggi Ed., Manduria: 87-127.

Castellana, M., 2014, "Il tetraedro storico-epistemologico", in Enriques, F.-Metzger, H., *Storia e struttura del pensiero scientifico*, Barbieri-Selvaggi Ed., Manduria: 117-145.

Castellana, M., 2015, "Gaston Bachelard ou la reverie anagogique dans les enjeux du surrationnel", "*Revue de synthèse*", t. 136: 93-116.

Castellana, M., 2015, "Federigo Enriques et la méthode historico-critique", "*Kairos. Journal of Philosophy of Science*", 14: 75-116.

Castellana, M., 2017, "Sur une petite phrase de Riemann'. Aspects du débat français autour de la Reasonable Effectiveness of Mathematics", " *Revue de synthèse*", t. 138: 195-229.

Castellana, M., 2018, "For an epistemology of mathematical contens: Albert Laut-

man", "*Lettera Matematica*", International Springer: 1-10.
Cesi, F, 2003, *Il natural desiderio di sapere. The Naural Desire for Knowledge*, Pontificia Academia Scientiarum, Vatican City.
Chaitin, G., 2007, *Alla ricerca di omega*, tr. it., Adelphi, Milano.
Chiffi, D., 2012, *Kurt Gödel. Philosophical explorations*, Aracne Ed., Roma.
Darboux, G., 1912, *Éloges académiques et discours*, Hermann, Paris.
Davis, M., 2005, "What Did Gödel Believe and When Did He Believe it?", "*The Bulletin of Symbolic Logic*", vol. 11: 194-206.
Desanti, J-T., 1968, *Les idéalités mathématiques*, Le Seuil, Paris.
Elkana, Y., 1995, *La rivoluzione scientifica come rivoluzione nella riflessività*, in La Vergata A.-Pagnini A., (cur.), *Storia della filosofia. Storia della scienza*, La Nuova Italia, Firenze.
Enriques, F., 1922, *Per la storia della logica*, Zanichelli, Milano.
Enriques, F., 1986[2], *Scienza e razionalismo*, Zanichelli, Bologna.
Geymonat, L., 1960, *Filosofia e filosofia della scienza*, Feltrinelli, Milano.
Føllesdal, D., 1995, in K. Gödel, *Collected Works, Vol. III: Unpublished Essays and Lectures,* Oxford University Press, Oxford: 362-365.
Girard, J.-Y., 2006, *Le point aveugle. Cours de logique. Vers la perfection*, Hermann, Paris.
Gödel, K., 1995, *Collected Works, Vol. III: Unpublished Essays and Lectures*, Oxford University Press, Oxford.
Gödel, K., 2013, *Collected Works, Vol. IV: Selected Correspondance A-G,* Oxford University Press, Oxford.
Gómez Dávila, N., 2001, *In margine a un testo implicito*, tr. it., Adelphi, Milano.
Gonseth, F., 1941, *Les Entretiens de Zurich sur les fondements et la méthode des sciences mathématiques*, Leeman, Zurich.
Guitart, P., 2015, "Deux problems en vue d'une épistémologie transitive des mathématiques", "*Revue de* synthèse", t. 136: 237-279.
Hintikka, J., 2000, *On Gödel*, Wadsworth, Belmont.
Ienna, G., 2016, *Presentazione* to Bachelard, G. *Metafisica della matematica*, tr. it., Castelvecchi, Roma, 5-18.
Lautman, A., 2006, *Essai sur l'unité des mathématiques dans leur development actuel*, Vrin, Paris.
Lautman. A., 2006, *Essai sur les notions de structure et d'existence en mathématique*, Vrin, Paris.
Lautman, A., 2017, *La matematica come resistenza*, tr. it. by M. Castellana, Castelvecchi, Roma.
Lecourt, D., 1997, *L'epistemologia storica di Gaston Bachelard*, in Canguilhem, G.-Lecourt, D., *L'epistemologia di Gaston Bachelard*, tr. by F. Bonicalzi, Jaca Book, Milano.
Lecourt, D., 2005, "La philosophe dans les sciences", "Revue de synthèse", t. 126: 451-454.
Lolli, G., 1992, *Incompletezza. Saggio su Kurt Gödel,* Il Mulino, Bologna.
Lolli, G., 2004, *Da Euclide a Gödel*, Il Mulino, Bologna.

Lolli, G. 2007, *Sotto il segno di Gödel*, Il Mulino, Bologna.
Martin, D.A., 2005, "Gödel's Conceptual Realism", "*The Bulletin of Symbolic Logic*", vol. 11: 207-224.
Metzger, H., 2014, *La scienza, l'appello alla religione e la volontà umana*, tr. it., by M. Castellana, Pensa Multimedia-ENS 'Pensée des sciences'.
Metzger, H., 2014, *Il metodo filosofico nella storia delle scienze*, in Enriques F.-Metzger H., Storia *e struttura del pensiero scientifico*, Barbieri-Selvaggi Ed. Manduria.
Patras, F., 2001, *La pensée mathématique contemporaine*, P.U.F. Paris.
Palombi, F., 2017, *L'elogio dell'astrazione. Gaston Bachelard e la filosofia della matematica*, Mimesis, Milano.
Petitot, J., 1995, *Pour un platonisme transcendantale*, in Panza M-Salankis J.M, (eds.), *L'objectivité mathématique. Platonisme et structures formelles*, Masson, Paris: 147-172.
Rodriguez-Consuegra, F., 1992, "Un inédito de Gödel contra el convencionalismo: historia, anàlisis, context y traducion", "*Arbor*", CXLII: 323-348.
Rodriguez-Consuegra, F., 1995, *Kurt Gödel. Unpublished Philosophical Essays*, Birkäuser Verlag, Basel-Boston- Berlin.
Schlick, M., 1986, *Teoria generale della conoscenza*, tr. it., F. Angeli Ed. Milano.
Vinti, C., 1997, *Il soggetto qualunque. Gaston Bachelard fenomenologo della soggettività epistemica,* Edizioni Scientifiche Italiane, Napoli.
Vinti, C., 2005, "Présence de Ferdinand Gonseth dans la pensée de Gaston Bachelard", "*Revue de synthèse*", t. 126: 391-415.
Vinti, C., 2014, *Gaston Bachelard, une épistémologie du sujet*, Mimesis, Milano.
Wallace, D.F., 2005, *Tutto, e di più. Una storia compatta dell'infinito*, tr. it. Codice, Torino.
Weyl, H., 1922, *Temps, espace, matière*, tr. French, Blanchard, Paris.
Widmar, B., 2017, *L'epistemologia,* by M. Castellana, Ed. Milella, Lecce.
Winter, M., 1911, *La méthode dans la philosophie des sciences*, Alcan, Paris.
Zalamea, F., 2018, *La philosophie synthétique des mathématiques*, tr. fr. by C. Alunni, Hermann, Paris.